

Calculating Acceleration

PowerPoint 9.2

Velocity-Time Graphs

A velocity-time graph illustrates an object's $\Delta \vec{v}$, which is the object's \vec{a} .

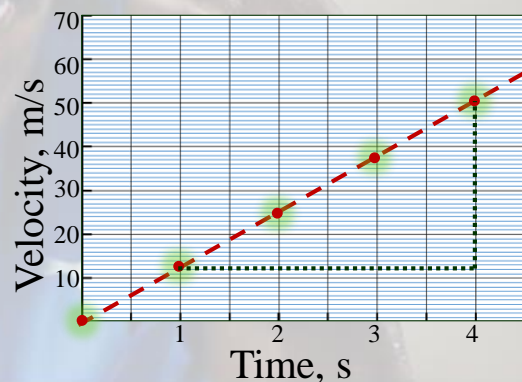
Time (s)	0.0	1.0	2.0	3.0	4.0
Velocity (m/s)	0.0	12.5	25.0	37.5	50.0

$$\text{Slope} = \frac{\Delta \vec{v}}{\Delta t} = \vec{a}$$

$$= \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$= \frac{50 \frac{m}{s} - 12.5 \frac{m}{s}}{4 s - 1 s} = 12.5 \frac{m}{s^2}$$

Units for acceleration



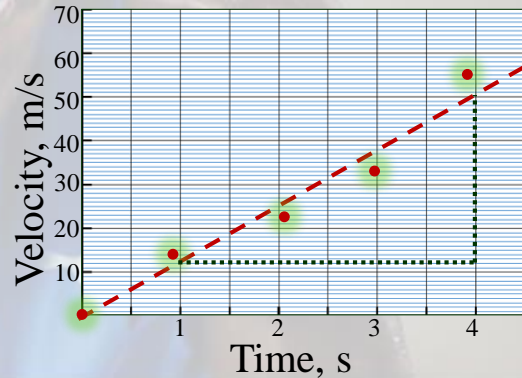
Velocity-Time Graphs

If \vec{a} is constant, then $\Delta\vec{v}$ will be the same over identical Δt .

In reality, constant acceleration is rare and a best-fit line is drawn in order to determine an object's **average acceleration**, \vec{a}_{av} .

➤ A straight line on a velocity-time graph is **constant acceleration**.

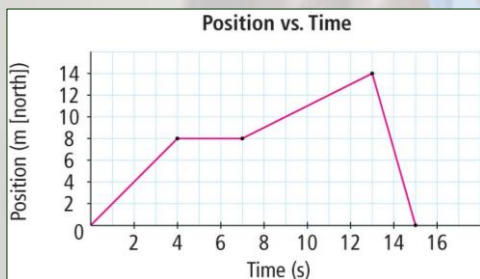
$$\begin{aligned}\vec{a}_{av} &= \frac{\Delta\vec{v}}{\Delta t} \\ &= \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} \\ &= \frac{50 \frac{m}{s} - 12.5 \frac{m}{s}}{4 s - 1 s} = 12.5 \frac{m}{s^2}\end{aligned}$$



Comparison Between Graphs

Position-Time Graphs, from 8.2

- Useful for objects at constant velocities
- *Slope* = \vec{v}_{av}



Velocity-Time Graphs

- Useful for objects whose velocity is not-constant
- Shows changes in velocity and acceleration of an object
- *Slope* = \vec{a} or \vec{a}_{av}

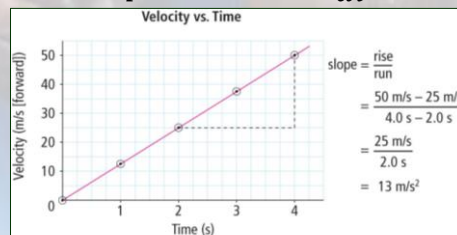


Figure 9.11 The slope of a velocity-time graph is the average acceleration of the object.

Determining Motion from a Velocity-Time Graph

0 to t_1

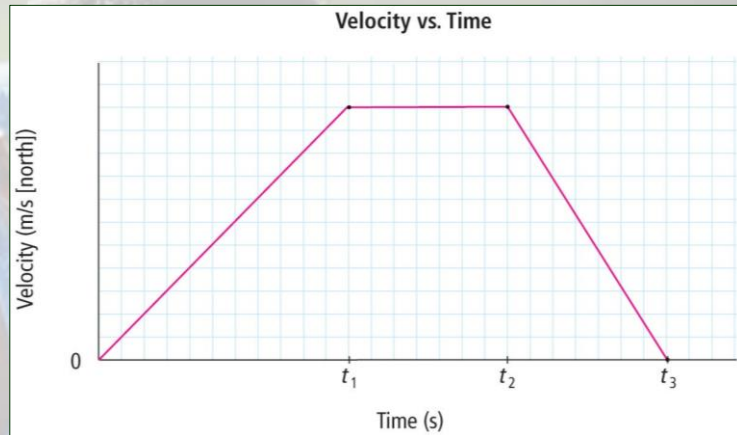
Constant positive \vec{a} ,
in a northerly direction

t_1 to t_2

$\vec{a} = 0$,
travelling at a constant \vec{v}

t_2 to t_3

Constant negative \vec{a} ,
in a southerly direction



0

t_1

t_2

t_3

Practice Calculating \vec{a} from a Velocity-Time Graph

0 s to 2 s

$$\vec{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\left(6 \frac{m}{s}\right) - \left(12 \frac{m}{s}\right)}{(2 s) - (0 s)} = -3 \frac{m}{s^2}$$

2 s to 4 s

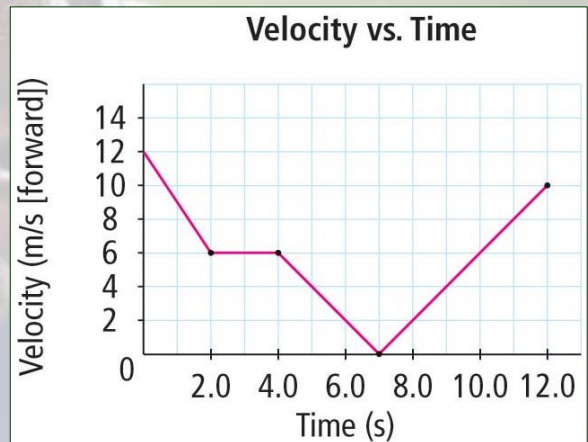
$$\vec{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\left(6 \frac{m}{s}\right) - \left(6 \frac{m}{s}\right)}{(4 s) - (2 s)} = 0 \frac{m}{s^2}$$

4 s to 7 s

$$\vec{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\left(0 \frac{m}{s}\right) - \left(6 \frac{m}{s}\right)}{(7 s) - (4 s)} = -2 \frac{m}{s^2}$$

7 s to 12 s

$$\vec{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\left(10 \frac{m}{s}\right) - \left(0 \frac{m}{s}\right)}{(12 s) - (7 s)} = 2 \frac{m}{s^2}$$



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Try Calculating \vec{a} without a Graph

Question

The ball's velocity changes from 2.5 m/s toward the cushion to 1.5 m/s away from the cushion in a time interval of 0.20 s.

Answer

$$\begin{aligned}\vec{a} &= \frac{\Delta\vec{v}}{\Delta t} \\ &= \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} \\ &= \frac{\left(-1.5 \frac{m}{s}\right) - \left(2.5 \frac{m}{s}\right)}{0.20 \text{ s}} = -20 \frac{m}{s^2}\end{aligned}$$

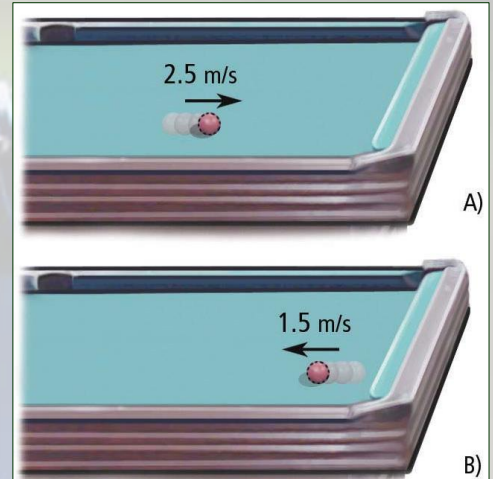


Figure 9.13 The ball's velocity changes from 2.5 m/s toward the cushion (A) to 1.5 m/s away from the cushion (B) in a time interval of 0.20 s.

Calculating $\Delta\vec{v}$ and Δt

The formula for \vec{a} can also be rewritten to calculate $\Delta\vec{v}$ or Δt .

$$\Delta\vec{v} = \vec{a}\Delta t \quad \leftarrow \quad \vec{a} = \frac{\Delta\vec{v}}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{\Delta\vec{v}}{\vec{a}}$$

Remembering the formula $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$, one can also calculate initial and final velocity.

Try this Practice Problem

Question

A motorcycle is travelling north at 11 m/s. How much time would it take for the motorcycle to increase its velocity to 26 m/s [N] if it accelerated at 3.0 m/s²?

Answer

Be sure to write this part down.

1. Write down what you know
2. Identify what you're asked to find.
3. Find formulae which will help you find what you're looking for and that contain values that are known.

$$1. \vec{a} = 3.0 \frac{m}{s^2} \quad 2. \Delta t = ? \quad 3. \Delta t = \frac{\Delta \vec{v}}{\vec{a}} = \frac{\left(15 \frac{m}{s}\right)}{3.0 \frac{m}{s^2}} = 5 s$$

$$v_i = 11 \frac{m}{s}$$

$$v_f = 26 \frac{m}{s}$$

$$\Delta \vec{v} = \vec{v}_f - \vec{v}_i = \left(26 \frac{m}{s}\right) - \left(11 \frac{m}{s}\right) = 15 \frac{m}{s}$$

Try this Practice Problem

Question

A skier moving 6.0 m/s forward begins to slow down, accelerating at -2.0 m/s² for 1.5 s. What is the skier's velocity at the end of the 1.5 s?

Answer

1. Write down what you know.
2. Identify what you're asked to find.
3. Find formulae which will help you find what you're looking for and that contain values that are known.

$$1. \vec{a} = -2.0 \frac{m}{s^2} \quad 2. \vec{v}_f = ? \quad 3. \Delta \vec{v} = \vec{v}_f - \vec{v}_i \Rightarrow \left(-3.0 \frac{m}{s}\right) = \vec{v}_f - \left(6.0 \frac{m}{s}\right)$$

$$\Delta t = 1.5 s$$

$$\vec{v}_i = 6.0 \frac{m}{s}$$

$$\Delta \vec{v} = \vec{a} \Delta t = \left(-2.0 \frac{m}{s^2}\right) (1.5 s) = -3.0 \frac{m}{s}$$

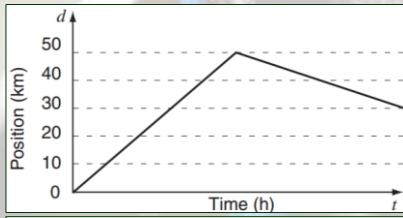
$$\vec{v}_f = 3.0 \frac{m}{s}$$

Provincial Exam Question

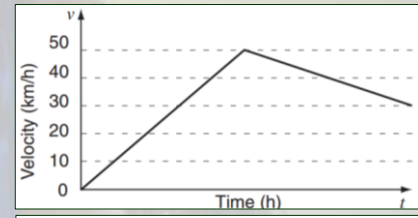
Question

Which of the following graphs shows a car travelling at a constant velocity of +50 km/h, then slowing down to +30 km/h as it enters a school zone?

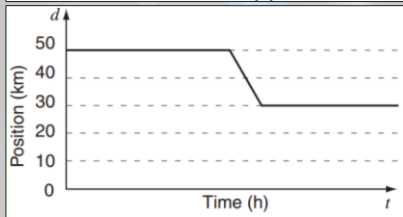
A.



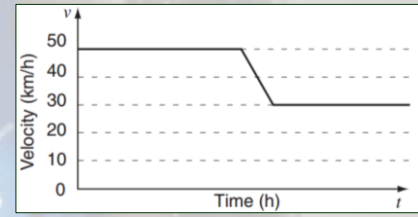
B.



C.



D.



Answer

D.

Provincial Exam Question

Question

On the surface of the moon, the acceleration due to gravity is -1.6 m/s^2 . If a ball is thrown upward at $+20 \text{ m/s}$, how long does it take for the ball to reach a velocity of 0 m/s before falling back to the surface of the moon?

A. 0.08 s

B. 12.5 s

C. 20 s

D. 32 s

Answer

$$\text{B. } 1. \vec{a} = -1.6 \frac{\text{m}}{\text{s}^2} \quad 2. \Delta t = ? \quad 3. \Delta t = \frac{\Delta \vec{v}}{\vec{a}} = \frac{(-20 \frac{\text{m}}{\text{s}})}{(-1.6 \frac{\text{m}}{\text{s}^2})} = 12.5 \text{ s}$$

$$\vec{v}_i = +20 \frac{\text{m}}{\text{s}}$$

$$\vec{v}_f = 0 \frac{\text{m}}{\text{s}}$$

$$\Delta \vec{v} = \vec{v}_f - \vec{v}_i = \left(0 \frac{\text{m}}{\text{s}}\right) - \left(20 \frac{\text{m}}{\text{s}}\right) = -20 \frac{\text{m}}{\text{s}}$$

Provincial Exam Question

Question

A car moving at +15 m/s pulls onto the highway and accelerates at +1.2 m/s² for 8.2 seconds. What is its final velocity?

- A. +9.8 m/s
- B. +16.2 m/s
- C. +20.7 m/s
- D. +24.8 m/s

Answer

D.

$$1. \vec{a} = +1.2 \frac{m}{s^2} \quad 2. \vec{v}_f = ? \quad 3. \Delta\vec{v} = \vec{v}_f - \vec{v}_i \rightarrow \left(9.84 \frac{m}{s}\right) = \vec{v}_f - \left(15 \frac{m}{s}\right)$$

$$\vec{v}_i = +15 \frac{m}{s} \qquad \qquad \qquad \vec{v}_f = 24.8 \frac{m}{s}$$

$$\Delta t = 8.2 s \qquad \qquad \qquad \Delta\vec{v} = \vec{a}\Delta t = \left(1.2 \frac{m}{s^2}\right)(8.2 s) = 9.84 \frac{m}{s}$$

Summary

Velocity-time graphs allow for analysis of changes in velocity over time. The slope of the best-fit line in such a graph is an object's **acceleration**.

With the formulae below, one can calculate the initial velocity, \vec{v}_i , the final velocity, \vec{v}_f , the time interval, Δt , and the acceleration, \vec{a} , of an object in motion.

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i \qquad \Delta\vec{v} = \vec{a}\Delta t \qquad \vec{a} = \frac{\Delta\vec{v}}{\Delta t} \qquad \Delta t = \frac{\Delta\vec{v}}{\vec{a}}$$